Sains Malaysiana 40(12)(2011): 1455–1459

On the Instability of Solutions of Nonlinear Delay Differential Equations of Fourth and Fifth Order

(Kestabilan Penyelesaian Persamaan Pembezaan Tunda Tak Linear Tertib ke Empat dan Lima)

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ABSTRACT

The main purpose of this paper is to introduce some new instability theorems related to certain fourth and fifth order nonlinear differential equations with a constant delay. By means of the Lyapunov-Krasovskii functional approach, we obtained two new results on the topic.

Keywords: Delay differential equation; fourth and fifth order; instability; Lyapunov-Krasovskii functional

ABSTRAK

Objektif utama artikel ini ialah untuk memperkenalkan beberapa teorem ketakstabilan yang baru berkaitan dengan persamaan pembezaan tak linear tertib ke empat dan lima dengan tundaan malar. Melalui pendekatan fungsi Lyapunov-Krasovskii, dua keputusan baru dalam topic ini telah diperoleh.

Kata kunci: Fungsi Lyapunov-Krasovskii; ketakstabilan; persamaan pembezaan tunda; tertib ke empat dan lima

INTRODUCTION

The qualitative theory of nonlinear differential equations of higher order has wide applications in science and technology (Chlouverakis & Sprott 2006; Linz 2008). In particular, by now, there are many works concerning the instability of solutions of certain, second, fourth and fifth order nonlinear differential equations without delay (Dong & Zhang 1999; Ezeilo1978a; Ezeilo1978b; Ezeilo1979a; Ezeilo1979b; Ezeilo 2000; Hale 1965; Li & Duan 2000; Li & Yu 1990; Lu & Liao 1993; Sadek 2003; Skrapek 1980; Sun & Hou 1999; Tiryaki 1988; Tiryaki 1999; Tunç 2004a; Tunç 2004b; Tunç 2005; Tunç 2006; Tunç 2008; Tunç 2009; C. Tunç & E. Tunç 2004; Tunç & Erdoğan 2007; Tunç & Karta 2008; Tunç & Şevli 2005) and the references cited thereof. It should be also noted that throughout all of the foregoing papers, based on Krasovskii's properties (see Krasovskii 1963), the Lyapunov's second (or direct) method has been used as a basic tool to prove the results established therein. On the other hand, it is crucial to obtain information on the qualitative behaviors of solutions of differential equations while we have no analytical expression for solutions. For this purpose, the theory of Lyapunov functions and functionals is a global and the most effective approach toward determining qualitative behaviors of solutions of higher order nonlinear differential equations.

It is worth mentioning that in (Èl'sgol'ts 1966) expresses " The formulation and proof of the Lyapunov's three theorems is almost unchanged if the system of differential equations without deviating arguments is changed to the system of differential equations with deviating arguments".

Meanwhile, Ezeilo (2000), and Ezeilo (1978b) established two instability results for the fourth and fifth order nonlinear differential equations without delay:

$$x^{(4)} + \psi(x'')x''' + g(x, x', x'', x''')x'' + \theta(x') + f(x) = 0 \tag{1}$$

and

$$x^{(5)} + \psi(x'')x''' + \phi(x'') + \theta(x') + f(x) = 0, \tag{2}$$

respectively.

In this paper, instead of (1) and (2), we consider the fourth and fifth order nonlinear delay differential equations:

$$x^{(4)} + \psi(x'')x''' + g(x, x(t-r), \ldots, x''', x'''(t-r))x'' + \theta(x') + f(x(t-r)) = 0, \tag{3}$$

and

$$x^{(5)} + \psi_1(x'')x''' + \phi(x, x(t-r), \ldots, x^{(4)}, x^{(4)}(t-r))x'' + \theta_1(x') + f_1(x(t-r)) = 0, \tag{4}$$

respectively.

We write (3) and (4) in system form as:

$x' = y$,

$y' = z$,

$z' = u,$
$u' = -\psi(z)u - g(x, x(t-r), \ldots, u, u(t-r))z - \theta(y) - f(x)$

$$+\int_{t-r}^{t} f'(x(s))y(s)\,ds, \qquad (5)$$

and

$x' = y,$
$y' = z,$
$z' = w,$
$w' = u,$
$u' = -\psi_1(z)w - \phi(x, x(t-r), \ldots, u, u(t-r))z - \theta_1(y) - f_1(x)$

$$+\int_{t-r}^{t} f_1'(x(s))y(s)\,ds, \qquad (6)$$

respectively, where r is a positive constant, fixed delay, the primes in (3) and (4) denote differentiation with respect to t, $t \in \Re_+ = [0,\infty)$; ψ, g, θ, f, ψ_1, ϕ, θ_1 and f_1 are continuous functions in their respective arguments on $\Re$, $\Re^8$, $\Re$, $\Re$, $\Re$, $\Re^{10}$, $\Re$ and $\Re$, respectively, and with $\theta(0) = \theta_1(0) = f(0) = f_1(0) = 0$, and satisfy a Lipschitz condition in their respective arguments. Hence, the existence and uniqueness of the solutions of (3) and (4) are guaranteed (Èl'sgol'ts 1966). We assume in what follows that the functions f and f_1 are also differentiable, and $x(t)$, $y(t)$, $z(t)$, $w(t)$ and $u(t)$ are abbreviated as x, y, z, w and u, respectively.

On the other hand, it is worth mentioning that to the best of our knowledge from the literature, we did not find any paper on the instability of solutions of fourth and fifth order delay differential equations. This paper is the first attempt on the topic. The basic reason with respect to the absence of any paper on this subject is due to the difficulty of the construction or definition of appropriate Lyapunov functionals for the instability problems concerning higher order delay differential equations. The construction and definition of appropriate Lyapunov functionals for higher order differential equations remain as a general problem in the literature. In this paper, by defining two appropriate Lyapunov functionals we verify our results. The motivation for this paper comes from the above mentioned papers and that of (Ko 1999). Our aim is to convey the results established in (Ezeilo 1978b) and (Ezeilo 2000) to nonlinear delay differential (3) and (4) for the instability of solutions of these equations.

Let $r \geq 0$ be given, and let $C = C([-r, 0], \Re^n)$ with:

$$\|\phi\| = \max_{-r\le s\le 0} |\phi(s)|, \phi \in C.$$

For $H > 0$ define $C_H \subset C$ by:

$$C_H = \{\phi \in C : \|\phi\| < H\}.$$

If $x:[-r, a] \to \Re^n$ is continuous, $0 < A \leq \infty$, then, for each t in $[0, A)$, x_t in C is defined by:

$$x_t(s) = x(t+s),\ -r \leq s \leq 0,\ t \geq 0.$$

Let G be an open subset of C and consider the general autonomous delay differential system with finite delay:

$$\dot{x} = F(x_t),\ x_t = x(t+\theta),\ -r \leq \theta \leq 0,\ t \geq 0.$$

where $F(0) = 0$ and $F{:}G \to \Re^n$ is continuous and maps closed and bounded sets into bounded sets. It follows from these conditions on F that each initial value problem:

$$\dot{x} = F(x_t),\ x_0 = \phi \in G,$$

has a unique solution defined on some interval $[0, A)$, $0 < A \leq \infty$. This solution will be denoted by $x(\phi)(.)$ so that $x_0(\phi) = \phi$.

Definition. *Let* $F(0) = 0$ *for* $t \geq 0$. *The zero solution*, $x = 0$, *of* $\dot{x} = F(x_t)$ *is stable if for each* $\varepsilon > 0$ *there exists* $\delta = \delta(\varepsilon) > 0$ *such that* $\|\phi\| < \delta$ *implies that* $|x(\phi)(t)| < \varepsilon$ *for all* $t \geq 0$. *The zero solution is said to be unstable if it is not stable.*

Main Results

Our first main result is the following theorem.

Theorem 1. *Together with all the assumptions imposed on the functions* ψ, g, θ *and* f *in Eq.* (3) *assume that there exist positive constants* a_2, a_4, ε *and* δ *such that the following conditions hold*:

$$f(0) = 0, f(x) \neq 0, (x \neq 0), \theta(0) = 0, \theta(y) \neq 0, (y \neq 0).$$
$$a_2 - g(x,x(t-r), y,y(t-r), z,z(t-r), u,u(t-r)) \geq \delta$$

for arbitrary $x, x(t-r), \ldots, u, u(t-r)$,

$$f'(x) - \frac{1}{4}a_2^2 > \varepsilon, f'(x) \leq a_4 \text{ for arbitrary } x.$$

Then, the zero solution, $x = 0$ *of Eq.* (3) *is unstable provided that*

$$r < \frac{2}{a_4}\min\{\varepsilon, \delta\}.$$

Remark. *The core of the proof of* Theorem 1 *will be to show that, under the conditions of* Theorem 1, *there exists a continuous functional* $V_1 = V_1(x_t, y_t, z_t, u_t)$ *which has the following three properties*:

(K_1) *In every neighborhood of* (0, 0, 0, 0) *there exists a point* (ξ, η, ζ, μ) *such that* $V_1(\xi, \eta, \zeta, \mu) > 0$,

(K_2) *the time* $\dot{V}_1 = \frac{d}{dt}V_1(x_t, y_t, z_t, u_t)$ *derivative along solution paths of the corresponding equivalent differential system for* Theorem 1 *is positive semi-definite*,

(K_3) *the only solution* $(x, y, z, u) = (x(t), y(t), z(t), u(t))$ *of* (5) *which satisfies is the trivial solution* (0, 0, 0, 0)

Proof. Consider the Lyapunov functional $V_1 = V_1(x_t, y_t, z_t, u_t)$ defined by

$$V_1 = \int_0^z \psi(s)sds + a_2yz + zu + yf(x)$$
$$+ \int_0^y \theta(s)ds - \lambda_1 \int_{-r}^{0} \int_{t+s}^{t} y^2(\theta)d\theta ds, \quad (7)$$

where s is a real variable such that the integral $\int_{-r}^{0}\int_{t+s}^{t} y^2(\theta)$ $d\theta ds$ is non-negative, and λ_1 is a positive constant which will be determined later in the proof.

It is clear that:

$$V_1(0, 0, \varepsilon_1^2, \varepsilon_1) = \varepsilon_1^3 - \frac{1}{2}\delta_1\varepsilon_1^4 > 0,$$

for all sufficiently small ε_1, $0<\varepsilon_1<1$, where $\delta_1 \equiv \max_{|z|\le 1}|\psi(z)|$. Hence, in every neighborhood of the origin, (0, 0, 0, 0) there exist a point $(0, 0, \varepsilon_1^2, \varepsilon_1)$ such that $V1(0, 0, \varepsilon_1^2, \varepsilon_1) > 0$, which verifies the property (K_1) (Krasovskii 1963).

The time derivative of $V_1(x_t, y_t, z_t, u_t)$ in (7) along with the solutions of (5) is given by:

$$\frac{d}{dt}V_1(x_t, y_t, z_t, u_t) = \left[a_2 - g(x, x(t-r), \ldots, u(t-r))\right]z^2$$
$$+\left[f'(x) - \frac{1}{4}a_2^2\right]y^2 + z\int_{t-r}^{t} f'(x(s))y(s)ds + \left[u + \frac{1}{2}a_2y\right]^2$$
$$-\lambda_1 ry^2 + \lambda_1\int_{t-r}^{t} y^2(s)ds. \quad (8)$$

Using the assumptions of Theorem 1 and applying the estimate $2|mn| \le m^2 + n^2$ one can easily get the following inequalities for the first three terms included in (8):

$$[a_2 - g(x,x(t-r)\ldots..u,u(t-r))]z^2 \ge \delta z^2,$$
$$\left[f'(x) - \frac{1}{4}a_2^2\right]y^2 \ge \varepsilon y^2,$$
$$z\int_{t-r}^{t} f'(x(s))y(s)ds \ge -|z|\int_{t=r}^{t} f'(x(s))|y(s)|ds$$
$$\ge -\frac{1}{2}\int_{t-r}^{t} f'(x(s))(z^2(t) + y^2(s))ds$$
$$\ge -\frac{a_4}{2}\int_{t-r}^{t}\{z^2(t) + y^2(s)\}ds$$
$$= -\frac{1}{2}a_4rz^2 - \frac{1}{4}a_4\int_{t-r}^{t} y^2(s)ds.$$

Then, the above inequalities imply:

$$\frac{d}{dt}V_1(x_t,y_t,z_t,u_t) \ge (\varepsilon-\lambda_1 r)y^2 + (\delta - 2^{-1}a_4r)z^2$$
$$+ (\lambda_1 - 2^{-1}a_4)\int_{t-r}^{t} y^2(s)ds.$$

Let $\lambda_1 = \frac{1}{2}a_4$. Hence

$$\frac{d}{dt}V_1(x_t,y_t,z_t,u_t) \ge (\varepsilon - 2^{-1}a_4r)y^2 + (\delta - 2^{-1}a_4r)z^2 > 0$$

provided that $r < \frac{2}{a_4}\min\{\varepsilon,\delta\}$, which verifies the property (K_2) (Krasovskii 1963).

On the other hand, $\frac{d}{dt}V_1(x_t,y_t,z_t,u_t) = 0$ if and only if $y = z = 0$, which implies that:

$$y = z = u = 0.$$

Besides, by $f(x) \ne 0$ for all $x \ne 0$ and the system (5), we can easily prove that $\frac{d}{dt}V_2(x_t,y_t,z_t,u_t)$ if and only if $x = y = z = u = 0$, which verifies the property (K_3) (Krasovskii 1963). By the foregoing discussion, we conclude that the zero solution of Eq. (3) is unstable.

The proof of Theorem 1 is now completed.

Example 1. Consider nonlinear fourth order delay differential equation of the form:

$$x^{(4)} + 5x''' + \left(1 + \frac{2}{1 + x^2 + x^2(t-r) + \ldots + u^2 + u^2(t-r)}\right)x''$$
$$+ 4x'(t) + 9x(t-r) = 0. \quad (9)$$

The above equation may be expressed in system form as follows:

$$x' = y,$$
$$y' = z,$$
$$z' = w,$$
$$w' = u,$$
$$u' = -5u - \left(1 + \frac{2}{1 + x^2 + \ldots + u^2(t-r)}\right)z$$
$$-4y - 9x + 9\int_{t-r}^{t} y(s)ds.$$

Clearly, Eq. (9) is special case of Eq. (3), and we have the following estimates:

$$\psi(z) = 5,$$
$$g(x,x(t-r),\ldots,u,u(t-r)) = 1 + \frac{2}{1 + x^2 + \ldots + u^2(t-r)}$$
$$3 - \frac{2}{1 + x^2 + \ldots + u^2(t-r)} \ge 1 = \delta,\ a_2 = 4,$$
$$\theta(y) = 4y,\ f(x) = 9x,\ f'(x) = 9 = a_4$$

and

$$f'(x) - \frac{1}{4}a_2^2 = 5 > \varepsilon = 1.$$

In view of the above discussion, it follows that all the assumptions of Theorem 1 hold. This shows that the zero solution $x = 0$ of Eq. (9) is unstable provided that $r < \frac{2}{9}$.

Our second main result is the following theorem.

Theorem 2. *Together with all the assumptions imposed on the functions* ψ_1, ϕ, θ_1 *and* f_1 *in* Eq. (4), *assume that there exist constants* a_3, $\bar{a}_5$ *and* a_5 *such that the following conditions hold*:

$f_1(0) = \theta_1(0) = 0, f_2(x) \neq 0, (x \neq 0), \bar{a}_5 \leq f'(x)\ a_5 < 0$ *for arbitrary x,*

$\theta_1(y) \neq 0, \ (y \neq 0),$

$\phi(x, x(t-r), y, y(t-r), z, z(t-r), w, w(t-r), u, u(t-r)) \geq a_3 > 0$

for arbitrary $x, x(t-r), \ldots, u, u(t-r)$.

Then, *the zero solution*, $x = 0$, *of Eq.* (4) *is unstable provided that*

$$r < \frac{2}{a_5}\min\{a_5, -a_3\}.$$

Proof. Consider now a Lyapunov functional $V_2 = V_2(x_t, y_t, z_t, w_t, u_t)$ defined by

$$V_2 = \frac{1}{2}w^2 - yf_1(x) - zu - \int_0^y \theta_1(s)ds - \int_0^z \psi_1(s)s ds - \lambda_2 \int_{-r}^{0}\int_{t+s}^{t} y^2(\theta)d\theta s, \tag{10}$$

where s is a real variable such that the integral is non-negative $\int_{-r}^{0}\int_{t+s}^{t} y^2(\theta)d\theta s$, and λ_2 is positive constant which will be determined later in the proof.

It is clear that

$$V_2(0, 0, 0, \varepsilon_2, 0) = \frac{1}{2}\varepsilon_2^2 > 0$$

for all sufficiently small ε_2. Hence, in every neighborhood of the origin, (0,0,0,0,0) there exist a point $(0,0,0,\varepsilon_2,0)$ such that $V_2(0,0,0,\varepsilon_2,0) > 0$, which shows that V_2 has the property (Krasovskii 1963).

On the other hand, by an elementary differentiation, time derivative of the functional $V_2(x_t,y_t,z_t,w_t,u_t)$ in (10) along the solutions of (6) yields that

$$\frac{d}{dt}V_2(x_t,y_t,z_t,w_t,u_t) = -f'(x)y^2 + \phi(x,x(t-r),\ldots,u,u(t-r))z^2$$
$$-z\int_{t-r}^{t} f'f'(x(s))y(s)ds - \lambda_2 r y^2 + \lambda_2\int_{t-r}^{t} y^2(s)ds.$$

The assumptions $\bar{a}_5 \leq f'(x) \leq a_5 < 0$, $\phi(.) \geq a_3 > 0$ and the estimate $2|mn| \leq m^2 + n^2$ imply that:

$$-z\int_{t-r}^{t} f'(x(s))y(s)ds \geq |z|\int_{t-r}^{t} f'(x(s))|y(s)|ds$$
$$\geq \frac{1}{2}\int_{t-r}^{t} f'(x(s))\{z^2(t) + y^2(s)\}ds$$
$$\geq \frac{\bar{a}_5}{2}\int_{t-r}^{t}\{z^2(t) + y^2(s)\}ds$$
$$= \frac{1}{2}\bar{a}_5 r z^2 + \frac{1}{2}\bar{a}_5\int_{t-r}^{t} y^2(s)ds.$$

so that:

$$\frac{d}{dt}V_2(x_t,y_t,z_t,w_t,u_t) = (-a_5 - \lambda_2 r)y^2 + (a_3 + 2^{-1}\bar{a}_5 r)z^2$$
$$+\left(\frac{1}{2}\bar{a}_5 + \lambda_2\right)\int_{t-r}^{t} y^2(s)ds.$$

Let $\lambda_2 = -\frac{1}{2}\bar{a}_5$. Hence

$$\frac{d}{dt}V_2(x_t,y_t,z_t,w_t,u_t) \geq \left(-a_5 + 2^{-1}\bar{a}_5 r\right)y^2 + \left(a_3 + 2^{-1}\bar{a}_5 r\right)z^2 > 0$$

provided that $r < \frac{2}{a_5}\min\{a_5, -a_3\}$, which verifies that V_2 has the property (K_2) (Krasovskii 1963).

On the other hand, $\frac{d}{dt}V_2(x_t,y_t,z_t,w_t,u_t) = 0$ if and only if $y = z = 0$, which implies that

$$y = z = w = u = 0.$$

Furthermore, by $f(x) \neq 0$ for all $x \neq 0$, it follows that $\frac{d}{dt}V_2(x_t,y_t,z_t,w_t,u_t) = 0$ if and only if $x = y = z = w = u = 0$. Thus, the property (K_3) is fulfilled by V_2 relative to (4) (Krasovskii 1963). By the foregoing discussion, we conclude that the zero solution of Eq. (4) is unstable. The proof of Theorem 2 is now completed.

Example 2. Consider nonlinear fifth order delay differential equation of the form

$$x^{(5)} + \left\{1 + (x'')^2\right\}x''' + \left\{1 + \exp\left(-1 - x^2 - x^2(t-r) - \ldots - u^2 - u^2(t-r)\right)\right\}x''$$
$$+ x'(t) - x(t-r) - 4arctgx(t-r) = 0. \tag{11}$$

We write Eq. (11) in system form as follows:

$$x' = y,$$
$$y' = z,$$
$$z' = w,$$
$$w' = u,$$
$$u' = -(1+z^2)w - \{1+\exp(-1-x^2-\ldots-u^2(t-r)\} - y + x + 4arctgx$$
$$-\int_{t-r}^{t} y(s)ds - 4\int_{t-r}^{t}\frac{1}{1+x^2(s)}y(s)ds.$$

It follows that Eq. (11) is special case of Eq. (4) and:

$\psi 1(z) = 1 + z^2,$

$\phi(x, x(t-r), \ldots, u, u(t-r)) = 1 + \exp\{-1 - x^2 - \ldots - u^2(t-r)\},$

$1 + \exp\{-1 - x^2 - \ldots - u^2(t-r)\} \geq 1 = a_3$

$\theta_1(y) = y, f_1(x) = -x - 4arctgx, f'(x) = -1 - 1\ \frac{4}{1+x^2}$

and

$$a_5 = -5 \leq -1- \frac{4}{1+x^2} \leq -1 = a_5.$$

In view of the above estimates, we conclude that all the assumptions of Theorem 2 hold. Hence, we arrive at the zero solution $x = 0$, (11) is unstable provided that $r < \frac{2}{5}$.

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Received: 9 December 2010
Accepted: 2 March 2011